Using the NAG Library to assist with mathematics research into diabetes modelling

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Researchers in numerical analysis tend to specialise in one single area such as linear algebra, differential equations ordinary or partial, deterministic or stochastic, integral/integro-differential equations, numerical integration, numerical approximation and optimization. They are supposed to either produce a new method which is better than existing ones in terms of accuracy, efficiency or stability, or to use/modify/extend an existing method to solve a new problem.

Very often the need arises to solve problems and sub-problems with specialized software. In these cases the researchers' efforts are supported by access to scientific software libraries. For example, in the area of diabetes modelling, a sub-problem of the numerical solution of integro-differential equations is the problem of linear or non-linear algebraic equations. This is then followed by the use of quadrature rules for the approximation of the resulting integrals.

A very well regarded scientific software library worldwide is the NAG Library from the Numerical Algorithms Group (NAG). In addition to supporting the work of researchers so that they work more efficiently in a competitive world, the NAG Library is a powerful tool for developing applications where it is critical to solve the given problem in an accurate and reliable way. It is also a valuable tool to use when supervising undergraduate or graduate work.

The author used the NAG Library at the University of Manchester when studying for her doctorate and subsequently as a lecturer. The NAG Library was also used, by the author, at Iowa State University for research and during the supervision of research based mini-projects assigned as part of a class on Scientific Computing. Various NAG routines have been used by the author at the University of Portsmouth during the supervision of B.Sc. theses on various subjects including integral equations and their applications in Actuarial Risk Management and Biology/Medicine.

Description of a simple numerical method applied to the Volterra integro-differential equation (VIDE)

$$\begin{aligned} y'(x) &= G(x, y(x), \int_0^x K(x,s,y(s))ds), 0 \le x \le X, \qquad (1) \\ y(0) &= y_0 \end{aligned}$$

where G, K are given functions sufficiently differentiable.

Letting

$$z(x) = \int_0^x K(x,s,y(s))ds, \qquad (2)$$

and integrating (1) from 0 to x, we obtain

$$y(x) = y_0 + \int_0^x G(t, y(t), z(t))dt, \qquad (3)$$

$$z(x) = \int_0^x K(x,s,y(s))ds. \qquad (4)$$

Consider the mesh points $x_i = ih, i = 0, 1, \ldots, n,\ h = X/n$. Discretizing (3),(4) at $x = x_i, i = 1, \ldots, n$ and applying the trapezoidal rule with weights $w_j, j = 0, 1, \ldots, i$ for the approximation of the integrals and denoting by y_i the approximation to $y(x_i)$, and by z_i the approximation to $z(x_i)$, we obtain

$$y_i = y_0 + h\sum_{j=0}^{i} w_j G(x_j, y_j, z_j) \qquad (5)$$

$$z_i = h\sum_{j=0}^{i} w_j K(x_i, x_j, y_j). \qquad (6)$$

For every $i = 1, 2, \ldots, n$ (5), (6) form a system of algebraic equations in y_i, z_i which in general is nonlinear.

More information about the numerical solution of Volterra integral and integro-differential equations can be found for example in the books by Linz(1985) and Brunner (2004).

Volterra integro-differential equations (VIDEs) have been used as models in many scientific areas including Engineering, Epidemiology, Biology and Medicine.

An area that the author is working in is that of the modelling of the glucose-insulin regulatory system and diabetes, with the ultimate aim to advance diabetic care and insulin delivery systems.

The model used (see panel) consists of an Ordinary Differential Equation and a VIDE and it was proposed by Mukhopadhyay, De Gaetano and Arino (2004) for analysing of the data (including estimation of parameters) obtained with an Intra Veinous Glucose Tolerance Test (IVGTT). The IVGTT is an experimental procedure in which a bolus amount of glucose is injected into an arm vein of an individual at rest, and then the plasma glucose and insulin concentrations are measured frequently over a period of usually 3 hours (cf. Mukhopadhyay, De Gaetano and Arino (2004), p. 408).

$$\begin{aligned} \frac{dG(t)}{dt} &= -b_1 G(t) - b_4 I(t) G(t) + b_7 \\ G(t) &= G_b, t \in (-\infty, 0) \\ \frac{dI(t)}{dt} &= -b_2 I(t) + b_6 \int_0^\infty w(s) G(t-s) ds \\ G(0) &= G_b + b_0 \\ I(0) &= I_b + b_3 b_0 \end{aligned} \qquad (7)$$

A $w(s)$ example:

$$w(s) = \alpha^2 s e^{-\alpha s}.$$

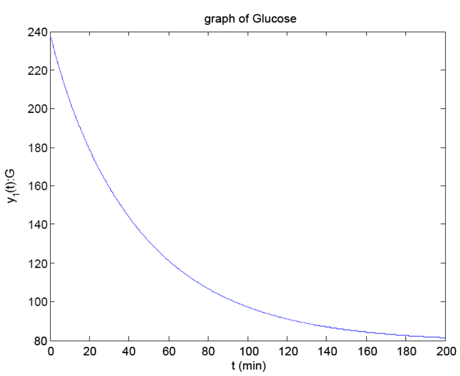
t [min] is time;
G [mg/dl] is the glucose plasma concentration;
G_b[mg/dl]is the basal (pre-injection) plasma glucose concentration;
I[μIU/ml] is the insulin plasma concentration;
I_b[μIU/ml] is the basal (pre-injection) insulin plasma concentration;
b_0 [mg/dl] is the theoretical increase in plasma concentration over basal glucose concentration at time zero after instantaneous administration and redistribution of the intravenous glucose bolus, (Mukhopadhyay, Gaetano and Arino (2004), p. 408-409).

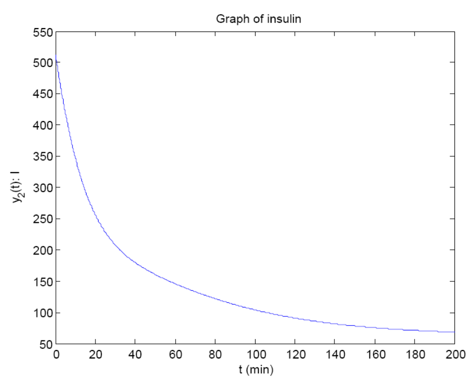
For more information about mathematical models and software used in diabetes modelling, we refer for example to the overview paper by Makroglou, Li and Kuang (2006).

The NAG libraries can be used to solve quickly this type of problem using routines in the D01 and D02 chapters. These routines can also be transferred easily to new programming and hardware platforms.

In addition to supporting traditional scientific computing programming languages, the availability of the NAG Toolbox for MATLAB has enriched the capabilities of this popular teaching and research software tool in many areas of Numerical Analysis. For example, in Numerical Integration, the MATLAB functions quad and quadl, are extended dramatically by the NAG Toolbox D01 chapter.

These graphs, plotted from MATLAB, give results from the solution of (7) using collocation methods (see also Makroglou and Karaoustas (2008)).





Currently, at the University of Portsmouth, the author is using NAG for the needs of a third year class titled Topics in Computational Mathematics. The well documented online help accompanied by examples facilitates the work of the instructor and makes its use easy for students.

References

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